Any set that represents the value of the Regular Expression is called a **Regular Set.**

Properties of Regular Sets

**Property 1**. *The union of two regular set is regular.*

**Proof** −

Let us take two regular expressions

RE1 = a(aa)\* and RE2 = (aa)\*

So, L1 = {a, aaa, aaaaa,.....} (Strings of odd length excluding Null)

and L2 ={ ε, aa, aaaa, aaaaaa,.......} (Strings of even length including Null)

L1 ∪ L2 = { ε, a, aa, aaa, aaaa, aaaaa, aaaaaa,.......}

(Strings of all possible lengths including Null)

RE (L1 ∪ L2) = a\* (which is a regular expression itself)

**Hence, proved.**

**Property 2.** *The intersection of two regular set is regular.*

**Proof** −

Let us take two regular expressions

RE1 = a(a\*) and RE2 = (aa)\*

So, L1 = { a,aa, aaa, aaaa, ....} (Strings of all possible lengths excluding Null)

L2 = { ε, aa, aaaa, aaaaaa,.......} (Strings of even length including Null)

L1 ∩ L2 = { aa, aaaa, aaaaaa,.......} (Strings of even length excluding Null)

RE (L1 ∩ L2) = aa(aa)\* which is a regular expression itself.

**Hence, proved.**

**Property 3.** *The complement of a regular set is regular.*

**Proof** −

Let us take a regular expression −

RE = (aa)\*

So, L = {ε, aa, aaaa, aaaaaa, .......} (Strings of even length including Null)

Complement of **L** is all the strings that is not in **L**.

So, L’ = {a, aaa, aaaaa, .....} (Strings of odd length excluding Null)

RE (L’) = a(aa)\* which is a regular expression itself.

**Hence, proved.**

**Property 4.** *The difference of two regular set is regular.*

**Proof** −

Let us take two regular expressions −

RE1 = a (a\*) and RE2 = (aa)\*

So, L1 = {a, aa, aaa, aaaa, ....} (Strings of all possible lengths excluding Null)

L2 = { ε, aa, aaaa, aaaaaa,.......} (Strings of even length including Null)

L1 – L2 = {a, aaa, aaaaa, aaaaaaa, ....}

(Strings of all odd lengths excluding Null)

RE (L1 – L2) = a (aa)\* which is a regular expression.

**Hence, proved.**

**Property 5.** *The reversal of a regular set is regular.*

**Proof** −

We have to prove **LR** is also regular if **L** is a regular set.

Let, L = {01, 10, 11, 10}

RE (L) = 01 + 10 + 11 + 10

LR = {10, 01, 11, 01}

RE (LR) = 01 + 10 + 11 + 10 which is regular

**Hence, proved.**

**Property 6.** *The closure of a regular set is regular.*

**Proof** −

If L = {a, aaa, aaaaa, .......} (Strings of odd length excluding Null)

i.e., RE (L) = a (aa)\*

L\* = {a, aa, aaa, aaaa , aaaaa,……………} (Strings of all lengths excluding Null)

RE (L\*) = a (a)\*

**Hence, proved.**

**Property 7.** *The concatenation of two regular sets is regular.*

**Proof −**

Let RE1 = (0+1)\*0 and RE2 = 01(0+1)\*

Here, L1 = {0, 00, 10, 000, 010, ......} (Set of strings ending in 0)

and L2 = {01, 010,011,.....} (Set of strings beginning with 01)

Then, L1 L2 = {001,0010,0011,0001,00010,00011,1001,10010,.............}

Set of strings containing 001 as a substring which can be represented by an RE − (0 + 1)\*001(0 + 1)\*

Hence, proved.

Identities Related to Regular Expressions

Given R, P, L, Q as regular expressions, the following identities hold −

* ∅\* = ε
* ε\* = ε
* RR\* = R\*R
* R\*R\* = R\*
* (R\*)\* = R\*
* RR\* = R\*R
* (PQ)\*P =P(QP)\*
* (a+b)\* = (a\*b\*)\* = (a\*+b\*)\* = (a+b\*)\* = a\*(ba\*)\*
* R + ∅ = ∅ + R = R (The identity for union)
* R ε = ε R = R (The identity for concatenation)
* ∅ L = L ∅ = ∅ (The annihilator for concatenation)
* R + R = R (Idempotent law)
* L (M + N) = LM + LN (Left distributive law)
* (M + N) L = ML + NL (Right distributive law)
* ε + RR\* = ε + R\*R = R\*